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Heterogeneous Consumption Goods, Sectoral Change and Economic Growth

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Abstract

This paper sets up a simple AK-type growth model with heterogeneous consumption goods. It is shown that the (overall) intertemporal elasticity of substitution, the saving rate, and the growth rate of income unambiguously increase in the course of economic development. Moreover, the model offers an intuitive explanation of sectoral change. It is demonstrated that there are a number of implications which are in line with the empirical evidence on economic growth and sectoral change.

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1 Introduction

The paper at hand sets up a simple AK-type growth model with heterogeneous consumption goods. The extension of the usual homogeneous consumption good framework yields a natural generalization with plausible implications and a number of new insights. More specifically, the paper adds to the literature on economic growth along three dimensions:

First, the model contributes to our understanding of the saving function. A thorough understanding of the saving function is important since endogenous growth models imply that an increase in the saving rate (investment rate) translates into a permanent acceleration of economic growth.¹

Second, Rebelo (1992) argues forcefully that standard growth models have serious difficulties in explaining (cross-country) differences in growth rates of consumption and income within an open economies framework. Most endogenous growth models explain differences in the growth rates with differences in the real rate of return to capital. As a result, perfect international capital markets (which equalize the real rate of return) would lead to identical growth rates of consumption (and GNP) across countries. The solution to this theoretical problem proposed by Rebelo is quite simple and intuitive. He employs an instantaneous utility function with subsistence consumption (Stone-Geary utility function), which gives rise to an increasing intertemporal elasticity of substitution (IES). As a result, the saving rate increases with the level of per capita income. In such a world, a unique rate of return does not cause economies to grow at a common rate simply because the saving rates diverge.

The paper at hand demonstrates that such a reasoning need not rely on subsistence consumption. It is shown that the (overall) intertemporal elasticity of substitution, the saving rate and the growth rate of income unambiguously increase with consumption once heterogeneity in consumer goods is taken into account.

Third, Kongsamut et al. (2001) employ a model with three different consumer goods and subsistence consumption to explain economic growth together

¹This relationship is confirmed empirically by Levine and Renelt (1992).

with sectoral change. Once more, the basic mechanism relies on subsistence consumption.² The model presented in this paper offers an alternative to reproduce the stylized facts on sectoral change as reported by Kuznets (1957).³

The paper is organized as follows: Section 2 introduces a fairly simple growth model with two heterogeneous consumption goods. Section 3 sets up the social planner's problem and derives the first-order conditions. In Section 4 the main implications are discussed. Section 5 provides a generalization to the case of n heterogeneous consumption goods. Finally, Section 6 offers a short summary together with some conclusions.

2 The model

A simple AK growth model with heterogeneous consumption goods is set up. To keep the analysis as simple as possible, only two types of consumer goods (x and y) are distinguished.⁴ Both goods are produced by employing a single input factor (capital) using constant returns to scale technologies according to:

$$x_s = A\theta k \quad (1)$$

$$y_s = B(1 - \theta)k, \quad (2)$$

where x_s and y_s denote the amount produced per unit of time, $A, B > 0$ are constant productivity parameters, $0 \leq \theta \leq 1$ is the share of capital allocated to x -production and k is the stock of (physical and human) capital at each instant of time. Good y is chosen as numeraire. Total output in units of y therefore is $q = p_x x_s + y_s$, where p_x is the price of x in units of y . Both goods can be used to add to the stock of capital. The equation of motion for capital

²In addition, the utility function employed by Kongsamut et al. (2001) is restricted in that the elasticities of marginal utility for the different goods must occur in fixed proportion.

³There are other papers which employ non-homothetic preferences to explain sectoral change. Foellmi and Zweimüller (2002) use a hierarchy-of-needs formulation (based on an explicit hierarchy function), while Meckl (2002) employs non-homothetic preferences similar to Kongsamut et al. (2001).

⁴Section 5 considers the generalization to the case of n goods.

accordingly reads:

$$\dot{k} = p_x(A\theta k - x) + B(1 - \theta)k - y. \quad (3)$$

where $\dot{k} := dk/dt$ and x and y denote the quantities consumed. Capital is assumed to be perfectly mobile across sectors and hence $p_x = B/A$. Therefore, the economy's budget constraint can be simplified to read:

$$\dot{k} = Bk - p_x x - y. \quad (4)$$

The above formulation illustrates that the production structure of this economy corresponds to a one-sector model. As a result, θ is not determined in this model.⁵

Intertemporal utility is assumed to be time separable:

$$U_0 = \int_0^\infty u(x, y) e^{-\rho t} dt, \quad (5)$$

where $\rho > 0$ is the time preference rate. Finally, instantaneous utility $u(x, y)$ is assumed to be additively separable across consumption goods:

$$u(x, y) = \frac{x^{1-\sigma} - 1}{1 - \sigma} + \frac{y^{1-\mu} - 1}{1 - \mu}, \quad (6)$$

where $\sigma, \mu > 0$ denote the respective constant elasticity of marginal utility with respect to consumption. The representative consumer perceives the two consumption goods as identical for $\sigma = \mu$. In this case, the utility function (6) is homothetic and the income-expansion path is linear.⁶ In contrast, the consumption goods are perceived differently for $\sigma \neq \mu$ and in this case the utility function (6) is non-homothetic implying a non-linear income-expansion path, as will be illustrated below (Section 4).⁷

⁵However, we can interpret $p_x x / (p_x x + y)$ as representing θ and $y / (p_x x + y)$ as representing $1 - \theta$.

⁶For $\sigma = \mu$, the utility function $\frac{x^{1-\sigma}}{1-\sigma} + \frac{y^{1-\mu}}{1-\mu}$ is homogenous. Hence, $\frac{x^{1-\sigma}-1}{1-\sigma} + \frac{y^{1-\mu}-1}{1-\mu}$ is homothetic.

⁷For $\sigma \neq \mu$, the utility function $\frac{x^{1-\sigma}}{1-\sigma} + \frac{y^{1-\mu}}{1-\mu}$ is non-homogenous and, hence, $\frac{x^{1-\sigma}-1}{1-\sigma} + \frac{y^{1-\mu}-1}{1-\mu}$ cannot be homothetic.

3 Dynamic problem and first-order conditions

The decentralized equilibrium and the Pareto-optimal solution coincide in this economy. The focus here is on the social planner's problem, which can be expressed as follows:

$$\max_{\{x,y\}} \int_0^\infty \left(\frac{x^{1-\sigma} - 1}{1-\sigma} + \frac{y^{1-\mu} - 1}{1-\mu} \right) e^{-\rho t} dt \quad (7)$$

$$s.t. \quad \dot{k} = Bk - p_x x - y \quad (8)$$

$$k(0) = k_0, \quad 0 < p_x x + y < Bk. \quad (9)$$

The associated (current-value) Hamiltonian function reads:

$$H = \frac{x^{1-\sigma} - 1}{1-\sigma} + \frac{y^{1-\mu} - 1}{1-\mu} + \lambda(Bk - p_x x - y) \quad (10)$$

and the necessary first-order conditions for interior solutions are given by:

$$H_x = x^{-\sigma} - \lambda p_x = 0 \quad (11)$$

$$H_y = y^{-\mu} - \lambda = 0 \quad (12)$$

$$\dot{\lambda} = -H_k + \rho\lambda = -B\rho + \rho\lambda \quad (13)$$

$$\dot{k} = H_\lambda = Bk - p_x x - y \quad (14)$$

$$\lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = 0. \quad (15)$$

In addition, the necessary conditions are also sufficient since the Hamiltonian is (jointly) concave in x , y and k (i.e. the Mangasarian sufficiency condition holds).

4 Implications

One can use (11) and (12) to eliminate x and y in (14). This gives a system of two differential equations in k and λ . Solving this system and taking (15) into account yields:

$$k = e^{\sigma^{-1}(B-\rho)t} \frac{p_x^{\frac{\sigma-1}{\sigma}} \sigma \lambda_0^{-\frac{1}{\sigma}}}{B(\sigma-1) + \rho} + e^{\mu^{-1}(B-\rho)t} \frac{\mu \lambda_0^{-\frac{1}{\mu}}}{B(\mu-1) + \rho} \quad (16)$$

$$\lambda = \lambda_0 e^{-(B-\rho)t}, \quad (17)$$

where λ_0 is implicitly determined by the following relation:

$$k_0 = \frac{p_x^{\frac{\sigma-1}{\sigma}} \sigma \lambda_0^{-\frac{1}{\sigma}}}{B(\sigma-1) + \rho} + \frac{\mu \lambda_0^{-\frac{1}{\mu}}}{B(\mu-1) + \rho}. \quad (18)$$

To describe the sectoral evolution of the economy under study, sectoral weights are defined according to:

$$w_x := \frac{p_x x}{p_x x + y} \quad \text{and} \quad w_y := \frac{y}{p_x x + y}, \quad (19)$$

which are completely determined by (11) and (12), (17) together with (18) and $k(0) = k_0$.

We are now in the position to discuss the main implications. At first, the instantaneous growth rate of overall output $q = p_x x_s + y_s$ is considered. Due to the simple one-sector structure of the model, overall output can be expressed as $q = Bk$ and hence $\hat{q} = \hat{k}$ (where $\hat{q} := \dot{q}/q$ etc.). Equation (16) immediately shows an important implication. The solution for k is a linear combination of two exponential functions. In the long run, the component with the larger root will dominate the pace of economic growth. Provided that $\sigma < \mu$, the asymptotic growth rate of capital (output) is hence given by $\lim_{t \rightarrow \infty} \hat{k} = \sigma^{-1}(B - \rho)$.⁸

Figure 1 (a) shows the time path of the growth rate of capital (output).⁹ Several issues are worth being noticed: (1) The growth rate is increasing over time. Therefore, the model implies (conditional) β -divergence. (2) The growth rate evolves within two boundaries: Assuming $\sigma < \mu$, the lower bound is $\mu^{-1}(B - \rho)$, while the upper bound is given by $\sigma^{-1}(B - \rho)$. (3) The transition period appears quite long with half-life of more than 100 years. This implication allows an explanation of diverging growth rates as transitional dynamics phenomenon.

⁸Also, for $\sigma = \mu$ equations (16) and (18) reduce to the familiar solution for the AK model with an index of homogeneous consumption goods to read $\hat{k} = \sigma^{-1}(B - \rho)$.

⁹The following set of parameters has been employed: $A = 0.2$, $B = 0.1$, $\rho = 0.05$, $\sigma = 1$, $\mu = 3$ and $k_0 = 1$.

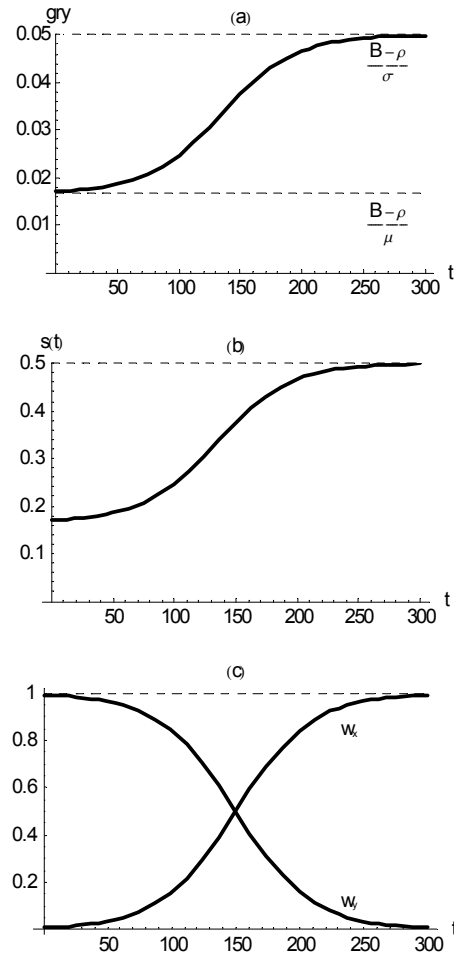


Figure 1: Transitional dynamics implications.

The economic intuition behind this pattern is easily explained by focusing on the (overall) IES. As is well known, along any optimal path the IES equals the sensitivity of the growth rate of consumption ($c = p_x x + y$) with respect to the difference between the real interest rate (B) and the time preference rate (ρ), i.e. $IES = \frac{\partial \hat{c}}{\partial (B-\rho)}$. Using (11) and (12) together with (17) one can readily derive the overall IES to read as follows:¹⁰

$$IES = w_x \sigma^{-1} + w_y \mu^{-1}. \quad (20)$$

¹⁰See also the proposition in Section 5.

The overall IES is obviously a weighted sum of the IES resulting from the subutility functions with the weights being equal to the respective consumption shares. Moreover, provided that $\sigma \neq \mu$ the consumption shares change as consumption rises. This is due to the fact that the income elasticities diverge in this case. More precisely, the income elasticities can be derived to read (for details see the appendix):¹¹

$$\varepsilon_{x,c} := \frac{\partial x}{\partial c} \frac{c}{x} = \frac{(x + x^{\frac{\sigma}{\mu}})\mu}{x\mu + x^{\frac{\sigma}{\mu}}\sigma} \quad (21)$$

$$\varepsilon_{y,c} := \frac{\partial y}{\partial c} \frac{c}{y} = \frac{(y + y^{\frac{\mu}{\sigma}})\sigma}{y\sigma + y^{\frac{\mu}{\sigma}}\mu}, \quad (22)$$

where c should in this context be interpreted as the point-in-time budget available for overall consumption. Table 1 shows the (asymptotic) value of the income elasticities and the relation between $\varepsilon_{x,c}$ and $\varepsilon_{y,c}$ (for finite values of x and y) depending on the relative values of σ and μ .

Table 1: Income elasticities

	$\sigma = \mu$	$\sigma < \mu$	$\sigma > \mu$
$\varepsilon_{x,c}$	1	$\lim_{x \rightarrow \infty} \varepsilon_{x,c} = 1$	$\lim_{x \rightarrow \infty} \varepsilon_{x,c} = \frac{\mu}{\sigma}$
$\varepsilon_{y,c}$	1	$\lim_{y \rightarrow \infty} \varepsilon_{y,c} = \frac{\sigma}{\mu}$	$\lim_{y \rightarrow \infty} \varepsilon_{y,c} = 1$
$\varepsilon_{x,c} \begin{matrix} \geq \\ \leq \end{matrix} \varepsilon_{y,c}$	$\varepsilon_{x,c} = \varepsilon_{y,c}$	$\varepsilon_{x,c} > \varepsilon_{y,c}$	$\varepsilon_{x,c} < \varepsilon_{y,c}$

In the case $\sigma = \mu$, we get $\varepsilon_{x,c} = \varepsilon_{y,c} = 1$. The underlying utility function implies an income-expansion path which is linear, as is illustrated by the left-hand panel of Figure 2. The overall IES is constant and equal to $\sigma^{-1} = \mu^{-1}$ from the beginning. As a result, the economy jumps immediately on the balanced growth path, i.e. there are no transitional dynamics. Moreover, for $p_x = 1$ the sectoral composition of consumption is $w_x = w_y = 0.5$.

For $\sigma \neq \mu$, we obtain diverging income elasticities. The underlying utility function implies an income-expansion path which is non-linear, as is illustrated

¹¹To simplify, p_x has been set equal to unity.

by the right-hand panel of Figure 2. As a result, the consumption structure changes as the economy grows. Equation (20) then shows that the overall IES varies. Let us consider the specific constellation $\sigma < \mu$. In this case, $\varepsilon_{x,c} > \varepsilon_{y,c}$ and hence w_x approaches unity, while w_y vanishes as the economy grows. Equation (20) shows that the overall IES increases and finally approaches σ^{-1} .

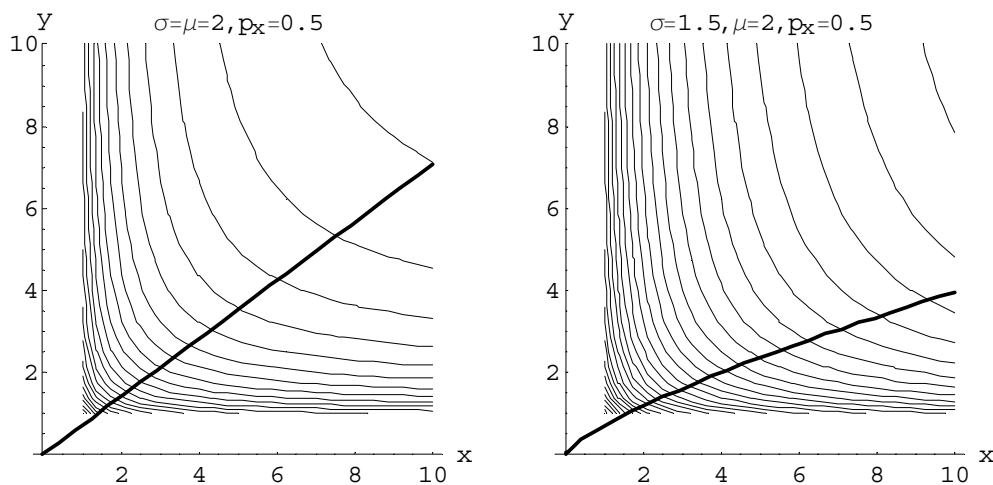


Figure 2: Illustration of alternative income expansion paths.

If, on the other hand, $\sigma > \mu$, we get $\varepsilon_{x,c} < \varepsilon_{y,c}$ and the consumption share w_y approaches unity, while w_x vanishes. The overall IES increases with consumption and approaches μ^{-1} .

It is important to notice that the overall IES increases in both cases. The rising overall IES causes the saving rate to increase also [Figure 1 (b)] and, as a consequence, growth accelerates [Figure 1 (a)]. Moreover, since the economy grows and converges very slowly initially, we observe substantial half-life of more than 100 years.

Let us now turn to the model's implication for sectoral change. The calibration of the model (as reported in footnote 10) implies an initial income elasticity of demand for x larger than unity and for y smaller than unity. Therefore, the consumption share of x increases, while the consumption share

of y decreases as the economy grows. The sectoral composition and its evolution over time are displayed in Figure 1 (c). It should be observed that w_x approaches unity, while w_y vanishes as time approaches infinity. More complex pattern of sectoral change may result once one considers more than two goods, as will be investigated in the next section.

5 The case of n goods

At this stage it is quite natural to consider the determination of the overall IES in the general case of n heterogeneous consumption goods. This information is provided by the following

Proposition:

Provided that the instantaneous utility function is additively separable with subutility functions which are characterized by constant elasticities of marginal utility:¹²

$$v(x_i) = \sum_{i=1}^n \frac{x_i^{1-\sigma_i} - 1}{1 - \sigma_i}, \quad (23)$$

the overall IES is given by:

$$IES = \sum_{i=1}^n \frac{p_i x_i}{\sum_{i=1}^n p_i x_i} \sigma_i^{-1}. \quad (24)$$

Proof:

In the case of n heterogeneous consumption goods, overall consumption is:

$$c = \sum_{i=1}^n p_i x_i, \quad (25)$$

where p_i denotes the price of good x_i (one p_i may be unity due to the choice of a numeraire). Differentiating (25) with respect to time and noting that prices are constant [due to the linear point-in-time production-possibility

¹²The proposition can be readily generalized to subutility functions with variable elasticities of marginal utility. In this case, we would write $v(x_i) = \sum_{i=1}^n u_i(x_i)$ with the elasticity of marginal utility expressed by $-\frac{\partial^2 u_i}{\partial x_i^2} x_i / \frac{\partial u_i}{\partial x_i}$.

frontier (PPF)] yields the growth rate of consumption to read:

$$\hat{c} = \sum_{i=1}^n \frac{p_i x_i}{\sum_{i=1}^n p_i x_i} \hat{x}_i. \quad (26)$$

Provided that instantaneous utility is additively separable in the n consumption goods, the n first-order conditions for the consumption goods can be written as:

$$x_i^{-\sigma_i} = p_i \lambda \quad \forall \quad i \in \{1, n\}. \quad (27)$$

By noting that a linear output technology implies $\hat{\lambda} = -(B - \rho)$, the growth rate of each consumption good may be expressed as:

$$\hat{x}_i = -\sigma_i^{-1} \hat{\lambda} = \sigma_i^{-1} (B - \rho) \quad \forall \quad i \in \{1, n\}. \quad (28)$$

Combining (26) and (28) immediately yields the growth rate of consumption to read:

$$\hat{c} = \sum_{i=1}^n \frac{p_i x_i}{\sum_{i=1}^n p_i x_i} \sigma_i^{-1} (B - \rho). \quad (29)$$

As already stated above, along any optimal path the (overall) IES equals the sensitivity of the consumption growth rate with respect to a change in the difference between the real rate of return and the time preference rate, i.e. $IES = \frac{\partial \hat{c}}{\partial (B - \rho)}$. Therefore, (29) proves the above stated proposition.

According to (24), the overall IES in the case of n heterogeneous consumption goods is given as a weighted sum of the IES from the subutility functions with the weights being equal to the respective consumption shares. The following points are worth being noted: (1) With heterogeneous consumption goods, the weights on the respective goods become important. The economic determinants of these weights are the income elasticities. (2) Provided that the income elasticities differ from unity such that the consumption structure changes in the course of economic development, a variable IES arises quite naturally. More specifically, since those goods with a high value of σ_i (a low IES) also experience a high consumption share for low levels of consumption, while those goods with a low value of σ_i (a high IES) experience a low share,

it is clear that the overall IES must increase in the course of economic development. (3) It is instructive to notice that the overall IES is not exclusively determined by preferences but is also affected by technology via the prices of the respective goods. To simplify matters, these have been assumed to be constant. However, it is clear that either a non-linear PPF or sector-specific technical change would exert an influence on goods prices and therefore on the overall IES.

We finally consider the example of a simple three goods economy. The instantaneous utility function then reads as follows:

$$u(x, y, z) = \frac{x^{1-\sigma} - 1}{1 - \sigma} + \frac{y^{1-\mu} - 1}{1 - \mu} + \frac{z^{1-\omega} - 1}{1 - \omega}, \quad (30)$$

where $\sigma, \mu, \omega > 0$.

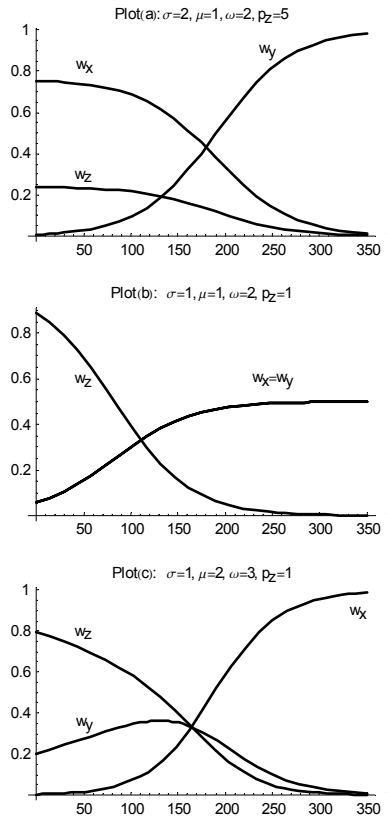


Figure 3: Illustration of alternative sectoral dynamics.

Two points can be learned from this little exercise. First, the implication according to which the consumption shares either approach unity or vanish [Figure 3, plot (a)] need not hold in this case.¹³ In contrast, it is possible that the consumption share of one good vanishes, while the consumption shares of the two other goods approach 0.5, as displayed in plot (b) of Figure 3.¹⁴

The second point concerns the issue of monotonic versus non-monotonic sectoral dynamics. It is easily demonstrated that this simple model with three consumption goods can explain non-monotonic sectoral dynamics, as illustrated by Figure 3, plot (c).

6 Summary and conclusion

The simple linear growth model with heterogeneous consumption goods yields a natural generalization of the usual homogenous consumption goods framework. This generalization leads to plausible implications and a number of new insights, which can be summarized as follows:

(1) It has been shown that an increasing overall IES arises naturally once heterogeneity in consumption goods is taken into account. This pattern is compatible with the empirical evidence on the relation between the IES and the level of wealth (e.g. Ogaki et al., 1996). This implication is further important since it shows that a rising IES is not restricted to the case of subsistence consumption in the utility function.

(2) The model implies (conditional) β -divergence. This growth pattern is compatible with both cross-sectional empirical evidence (e.g. Zind, 1991) as well as time-series evidence (e.g. Romer, 1986). The model describes a new mechanism of β -divergence. This mechanism results from an increase in the overall IES and the saving rate due to heterogeneity in consumption goods. As

¹³Of course, for $\sigma = \mu$ the consumption shares could also be constant from the beginning in the two-goods case.

¹⁴The following parameters are the same as before: $A = 0.2$, $B = 0.1$, $\rho = 0.05$ and $k_0 = 1$. Those parameters which have been altered are reported in the respective plots of Figure 3.

a consequence, the growth rate increases with the level of per capita income.

(3) The variability in the IES allows an explanation of different growth experiences in a world with integrated capital markets. As Rebelo (1992) has pointed out, standard endogenous growth models have serious difficulties in explaining diverging growth rates when economies face a unique world interest rate.

(4) The model also sheds some light on the "why doesn't capital flow from rich to poor countries" puzzle (Lucas, 1990). In the underlying model, there is a unique real rate of return. Poor countries have a different composition of demand and hence a lower saving rate. Since there is a unique real rate of return, there is no reason for capital to flow from rich to poor countries.

(5) The model can explain sectoral change driven by changes in the composition of consumption. In contrast to Kongsamut et al. (2001), the model does not rely on subsistence consumption. In addition, non-monotonicities in the sectoral composition of output can easily be explained. This is important when it comes to a theoretical explanation of the stylized facts on sectoral change (e.g. Kongsamut et al., 2001).

There are interesting extensions of the model which are left for future research. For instance, it would be clearly interesting to see the consequences of sector-specific technical change. However, introducing technical progress would yield a permanent acceleration of growth in the AK-type growth model employed in this paper. A neoclassical framework could be used with sector specific technical progress. One can expect that the transition process would be more complex and consequently a higher degree of non-monotonicity should be observed.

Colophon

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7 Appendix

7.1 Derivation of income elasticities: equations (21) and (22)

The Lagrangian function for the static maximization problem reads:

$$L = \frac{x^{1-\sigma} - 1}{1 - \sigma} + \frac{y^{1-\mu} - 1}{1 - \mu} + \xi(c - p_x x - y)$$

and the necessary first-order conditions are:

$$x^{-\sigma} - p_x \xi = 0$$

$$y^{-\mu} - \xi = 0$$

$$c - p_x x - y = 0.$$

From these first-order conditions, one can readily derive the (implicit) demand functions for x and y to read as follows:

$$c - p_x x - \left(\frac{x^{-\sigma}}{p_x} \right)^{-1/\mu} = 0 \quad (31)$$

$$c - p_x (p_x y^{-\mu})^{-1/\sigma} - y = 0. \quad (32)$$

Taking the definition of income elasticities $\varepsilon_{x,c} := \frac{\partial x}{\partial c} \frac{c}{x}$ and $\varepsilon_{y,c} := \frac{\partial y}{\partial c} \frac{c}{y}$ into account, implicitly differentiating (31) and (32) and simplifying gives the RHS of (21) and (22) in the main text.

7.2 Income elasticities (asymptotic properties): Table 1

Consider the income elasticities $\varepsilon_{x,c}$ and $\varepsilon_{y,c}$ as given by (21) and (22). Let us focus on (21) first. Slightly reformulating gives:

$$\varepsilon_{x,c} = \frac{(x + x^{\frac{\sigma}{\mu}})\mu}{x\mu + x^{\frac{\sigma}{\mu}}\sigma} = \frac{x\mu}{x\mu + x^{\frac{\sigma}{\mu}}\sigma} + \frac{x^{\frac{\sigma}{\mu}}\mu}{x\mu + x^{\frac{\sigma}{\mu}}\sigma} = \frac{1}{1 + \frac{x^{\frac{\sigma}{\mu}}\sigma}{x\mu}} + \frac{1}{\frac{x\mu}{x^{\frac{\sigma}{\mu}}\mu} + \frac{\sigma}{\mu}} \quad (33)$$

$$\varepsilon_{x,c} = \frac{1}{1 + x^{\frac{\sigma}{\mu}-1} \frac{\sigma}{\mu}} + \frac{1}{x^{1-\frac{\sigma}{\mu}} + \frac{\sigma}{\mu}} \quad (34)$$

This expression shows that: (1) $\varepsilon_{x,c} = 1$ for $\sigma = \mu$; (2) $\lim_{x \rightarrow \infty} \varepsilon_{x,c} = 1$ for $\sigma < \mu$ ($x^{\frac{\sigma}{\mu}-1}$ vanishes and $x^{1-\frac{\sigma}{\mu}}$ diverges); (3) $\lim_{x \rightarrow \infty} \varepsilon_{x,c} = \frac{\mu}{\sigma}$ for $\sigma > \mu$ ($x^{\frac{\sigma}{\mu}-1}$ diverges and $x^{1-\frac{\sigma}{\mu}}$ vanishes).

Similar reasoning can be conducted for $\varepsilon_{y,c}$ shown in (22):

$$\varepsilon_{y,c} = \frac{(y + y^{\frac{\mu}{\sigma}})\sigma}{y\sigma + y^{\frac{\mu}{\sigma}}\mu} = \frac{y\sigma}{y\sigma + y^{\frac{\mu}{\sigma}}\mu} + \frac{y^{\frac{\mu}{\sigma}}\sigma}{y\sigma + y^{\frac{\mu}{\sigma}}\mu} = \frac{1}{1 + \frac{y^{\frac{\mu}{\sigma}}\mu}{y\sigma}} + \frac{1}{\frac{y\sigma}{y^{\frac{\mu}{\sigma}}\sigma} + \frac{\mu}{\sigma}} \quad (35)$$

$$\varepsilon_{y,c} = \frac{1}{1 + y^{\frac{\mu}{\sigma}-1}\frac{\mu}{\sigma}} + \frac{1}{y^{1-\frac{\mu}{\sigma}} + \frac{\mu}{\sigma}} \quad (36)$$

Here one recognizes that: (1) $\varepsilon_{y,c} = 1$ for $\sigma = \mu$; (2) $\lim_{y \rightarrow \infty} \varepsilon_{y,c} = \frac{\sigma}{\mu}$ for $\sigma < \mu$ ($y^{\frac{\mu}{\sigma}-1}$ diverges and $y^{1-\frac{\mu}{\sigma}}$ vanishes); (3) $\lim_{y \rightarrow \infty} \varepsilon_{y,c} = 1$ for $\sigma > \mu$ ($y^{\frac{\mu}{\sigma}-1}$ vanishes and $y^{1-\frac{\mu}{\sigma}}$ diverges).

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